P.R. GOVERNMENT COLLEGE (A), KAKINADA

II B.Sc. – MATHEMATICS – Semester - III (w.e.f. 2017-2018)

Course: ABSTRACT ALGEBRA

Total Hrs. of Teaching Learning & Evaluation: 90 @ 6 h / Week Total Credits: 05

Objective:

- To learn about the basic structure in Algebra
- To understand the concepts and able to write the proofs to theorems
- To know about the applications of group theory in real world problems

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Unit I: Groups (16 Hrs)

Binary Operation – Algebraic structure – semi group – monoid – Definition and elementary properties of a Group – Finite and Infinite groups – Examples – Order of a group – Composition tables with examples.

Unit II: Subgroups, Cosets and Lagrange's Theorem (16 Hrs)

Definition of Complex – Multiplication of two complexes – Inverse of a complex – Subgroup definition – Examples - Criterion for a complex to be a subgroup – Criterion for the product of two subgroups to be a subgroup – Union and intersection of subgroups.

Cosets definition – Properties of cosets – Index of subgroup of a finite group – Lagrange's Theorem.

Unit III: Normal Subgroups (14 Hrs)

Definition of normal subgroup – Proper and improper normal subgroup – Hamilton group – Criterion for a subgroup to be a normal subgroup – Intersection of two normal subgroups – Subgroup of index 2 is a normal subgroup – Simple group – Quotient group – Criteria for the existence of a quotient group.

Unit IV: Homomorphism (13 Hrs)

Definition of homomorphism – Image of homomorphism – elementary properties of homomorphism – Definition and elementary properties of Isomorphism and automorphism – Kernel of a homomorphism – Fundamental theorem on homomorphism and applications.

Unit V: Permutations and Cyclic Groups (16 Hrs)

Definition of permutation – Permutation multiplication – Inverse of a permutation – Cyclic permutations – Transposition – Even and odd permutations – Cayley's theorem.

Definition of cyclic group - Elementary properties – Classification of cyclic groups.

Co-Curricular: Assignment, Seminar, Quiz, etc. **(15 Hrs)**

Additional Inputs: Applications of group theory

Text Book: Abstract Algebra by J.B.Fraleigh

Books for reference:

1. A text book of Mathematics, S. Chand and Company, Ltd.
2. Modern Algebra by Gupta and Malik.
3. Elements of Real Analysis by Santhi Nararayana & M. D. Raisinghania.

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-III

Unit	TOPIC	V.S.A.Q	S.A.Q	E.Q	Marks allotted to the Unit
I	Groups	1	1	2	22
II	Subgroups, Cosets & Lagrange's theorem	1	1	2	22
III	Normal Subgroups	1	1	1	14
IV	Homomorphism	1	1	1	14
V	Permutations and Cyclic groups	1	1	2	22
Total		5	5	8	94

V.S.A.Q. = Very short answer questions (1 mark)

S.A.Q. = Short answer questions (5 marks)

E.Q = Essay questions (8 marks)

Very short answer questions : 5 X 1 = 05

Short answer questions : 3 X 5 = 15

Essay questions : 5 X 8 = 40

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Total Marks = 60

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P.R. Government College (Autonomous), Kakinada
II year B.Sc., Degree Examinations - III Semester
Mathematics Course: Abstract Algebra
Paper III (Model Paper w.e.f. 2018 - 2019)

Time: 2Hrs 30 min **Max. Marks: 60**

PART –I

Answer the following questions. Each question carries 1 mark. **5 x 1 = 5 M**

1. Write the Cauchy's composition table for $G = \{1,\ \omega,\ \omega^2\}$.
2. Write a proper subgroup of a group $G = \{1,\ -1,\ i,\ -i\}$ with respect to multiplication.
3. Define normal subgroup.
4. Check whether $f: (Z,\ +) \longrightarrow (Z,\ +)$ defined by $f(x) = x^2$ is a homomorphism or not.
5. Write the inverse of the permutation $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 1 & 2 & 5 \end{pmatrix}$.

PART-II

Answer any THREE questions. Each question carries 5 marks. **3 x 5 = 15 M**

6. Prove that the set Z of all integers form an abelian group w.r.t. the operation defined by $a * b = a + b + 2, \forall\, a, b \in Z$.
7. Prove that a non empty complex H of a group G is a subgroup of G if and only if $H = H^{-1}$.
8. If M, N are two normal subgroups of G such that $M \cap N = \{e\}$ then every element of M commutes with every element of N.
9. If f is a homomorphism of a group G into a group G', then prove that the kernel of f is a normal subgroup of G.
10. Express the product $(2\quad 5\quad 4)\,(1\quad 4\quad 3)\,(2\quad 1)$ as a product of disjoint cycles and find its inverse.

PART-III

Answer any <u>FIVE</u> questions from the following by choosing at least <u>TWO</u> from each section. Each question carries 8 marks. **5 x 8 = 40 M**

SECTION-A

11. Show that the n^{th} roots of unity form an abelian group with respect to multiplication.
12. Prove that a semi group $(G, .)$ is a group if and only if the equations $ax = b$, $ya = b\ \forall\, a, b \in G$ have an unique solutions in G.
13. State and Prove the necessary and sufficient condition for a finite complex H of a group G to be a subgroup of G.
14. State and prove Lagrange's theorem.

SECTION-B

15. If H is a normal subgroup of a group (G, .), then prove that the product of two right (left) cosets of H is also a right (left) coset of H.
16. Prove that every homomorphic image of a group G is isomorphic to some quotient group of G.
17. Prove that the set A_n of all even permutations form a normal subgroup of the group of permutations S_n.
18. Prove that every subgroup of a cyclic group is cyclic.